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ANALYSIS OF A ROMAN PORTABLE DIAL

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The instrument shown in Fig. 1 is in the Museum of the History of Science in the University of Oxford[1] and is a fine example of a Roman portable sundial. Of the several authors who have written about it, three are of particular note: Rene Rohr,[2] Michael Wright[3] and Stephen Johnston.[4] Rohr gives a good introduction but without any analysis. Wright provides a historical account as well as a mathematical treatment. Johnston alone derives the simple relationship between indicated hour and solar altitude. It is noted below that the same relationship governs the operation of the horary quadrant.

Fig. 1. A Roman portable sundial. Photo by permission of the Museum of the History of Science, University of Oxford. MHS inv. 51358.

The present article introduces a novel construction which makes reasoning about the instrument very straightforward.

It is shown that, although there are scales for setting the latitude and the current solar declination, it is not strictly necessary to know either. Also, the instrument is much more universal than its designer seems to have appreciated.

Dial Components

The dial features an inner disc which sits in a recess in an outer disc whose external diameter is 61 mm. This is suspended by a length of cord. The most important component is the gnomon assembly, which includes the gnomon proper and a scale which incorporates engraved lines; these are hour lines that indicate unequal hours. An unequal hour is one-twelfth of the daylight period.

Simple Example – Measuring Solar Altitude

The gnomon assembly can be swivelled about a horizontal axis which runs through the centres of the discs. The schematic diagram in Fig. 2 shows the gnomon assembly in profile and arranged so that it is vertical.

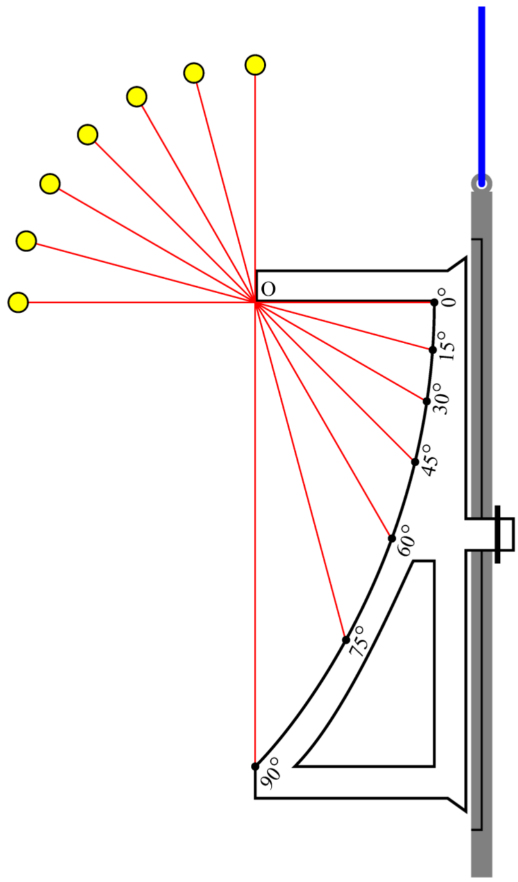

Fig. 2. The gnomon assembly seen in profile.

The gnomon is the finger at the top and O marks the edge between its two most important faces. The discs are assumed to be much heavier than the gnomon assembly so that, when the instrument is hanging freely from the blue cord, the discs (seen in cross-section) are vertical.

Dots mark the positions of the engraved hour lines on the curved surface and these dots are provisionally labelled with angles at 15° intervals ranging from 0° to 90°. The lines on the instrument itself are not labelled.

The instrument is classed as an altitude dial. It determines the unequal hour from the solar altitude. When the gnomon assembly is set as in Fig. 2, the instrument can measure solar altitude directly. The user first checks that the instrument is hanging vertically and then follows this golden rule:

Ensure that the sun is in the plane of the gnomon assembly.

The user has to twist the suspension cord so that the sun shines equally on the two cheeks of the gnomon assembly. The shadow of the edge of the gnomon at point O will then fall on the curved surface and, by interpolating between the two nearest dots, the user can determine the solar altitude.

In Fig. 2, schematic suns are shown at seven altitudes from 0° to 90° at 15° intervals and, from each sun, a red line runs to the relevant dot on the curved surface. In the diagram, the seven suns are shown in the same plane. In general, the azimuth of the sun changes continuously during the day and, at each observation, the user has to face the instrument in a different direction but, in following the golden rule, it is never necessary to know what that direction is.

During the period of daylight, the altitude of the sun is 0° at sunrise, then rises to a maximum at noon, before returning to 0° again at sunset. The maximum possible noon altitude is 90° and *in that special case* (and subject to two caveats) the arrangement in Fig. 2 can be used to determine the unequal hour by dividing the indicated altitude by 15. For example, if the altitude is 30° then the unequal hour is 2.

The first caveat, which applies to all altitude dials, is that the user needs to know whether it is morning or afternoon because the sun reaches each particular altitude twice during the course of a day. If the altitude is 30° and the sun has gone past its highest point then the unequal hour is 10 not 2. The result obtained by dividing by 15 has to be subtracted from 12.

The second caveat is that the underlying mathematical model is slightly flawed. When exactly two-twelfths of the day have elapsed, the solar altitude can be almost a degree below 30°, a small error.

Of course, a noon altitude of 90° is a special case. If the solar altitude at noon is, say, 40° then an observed altitude of 30° will not indicate that we are at hour 2 or at hour 10 but at some time much closer to noon.

In general, the user has to follow another rule:

> *Set the gnomon assembly so that its slope (relative to the horizontal) is the altitude of the sun at noon.*

In the case illustrated in Fig. 2, the sun has an altitude of 90° at noon so the gnomon assembly has to be vertical. If the solar altitude at noon is 40° then the slope of the gnomon assembly has to be 40° too.

A Modified Instrument – An Aid to Understanding

It is considerably easier to understand the operation of the instrument if the gnomon assembly is replaced by the quadrant construction shown in Fig. 3. The slope of the quadrant is the solar altitude at noon, taken here as 40°.

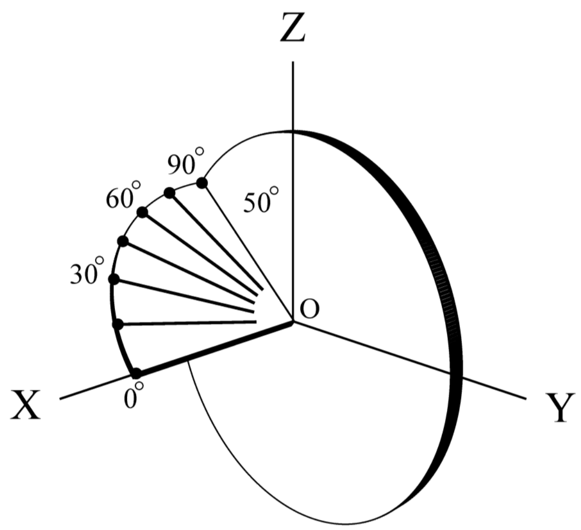

Fig. 3. The gnomon assembly replaced by a quadrant whose angle to the horizontal matches the solar altitude at noon, taken here as 40°. The angle to the vertical is 50°.

There is now a single vertical disc and the sloping edge of the quadrant is attached to a radius of that disc. The other edge of the quadrant is horizontal and perpendicular to the plane of the disc.

An X, Y, Z system of rectangular coordinates, with origin O, has been added for reference purposes. The X–Y plane is horizontal; the positive X–axis coincides with the horizontal edge of the quadrant and the Z–axis is vertical.

The origin O equates to point O in Fig. 2. Seven hour lines with dots at their outer ends are shown radiating from point O at 15° intervals ranging from 0° to 90°.

Now imagine a bug stationed at point O. When the bug sights along the 0° line it is looking at some point on the horizon; the altitude of the line is 0°. When the bug sights along the 90° line it is looking at some point in the sky whose altitude is 40°.

If the bug scans along every radius from the 0° line to the 90° line its gaze must span every altitude from 0° to 40° and this is exactly the range of altitudes of the sun on a day when the altitude at noon is 40°.

To determine the unequal hour, the user first ensures that the quadrant is in the plane of the sun (see Fig. 4) and then notes which radius on the quadrant aligns with the line from the sun to point O. (The bug will have no difficulty but, for humans, it is better to use the gnomon assembly!) The angular offset of this radius from the 0° line on the quadrant is divided by 15 and the result (subtracted from 12 if it is the afternoon) gives the unequal hour.

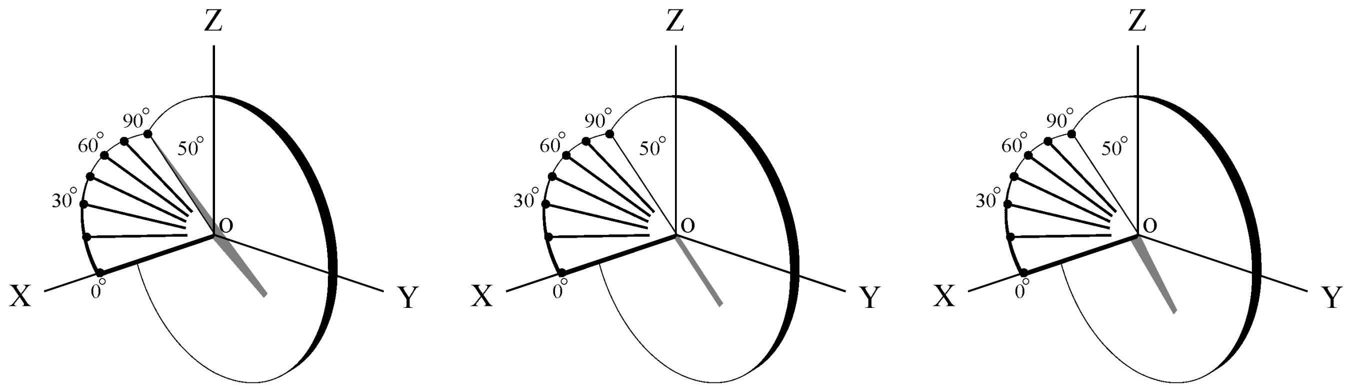

Fig. 4. In the central figure the shadow aligns with the sloping edge of the quadrant; the sun is shining equally on both faces of the quadrant as required. In the figure on the left the sun is shining on the lower face. In the figure on the right it is shining on the upper face. To correct these latter cases, the user has to rotate the supporting disc slightly about its vertical axis.

Clearly the altitude of the sun at hour 0 (sunrise) matches the altitude of the 0° line on the quadrant and the altitude at hour 6 (noon) matches the altitude of the 90° line.

There is an underlying assumption that this correspondence applies more generally, thus the altitude of the sun at unequal hour u [or $12-u$] matches the altitude of the radius which is offset by an angle of $15u°$ [or $15(12-u)°$] from the reference 0° radius on the quadrant. In practice the match may be imperfect but the error is usually small.

Inherent Asymmetry

One may imagine that first thing each morning an assiduous user would set the slope of the quadrant to the noon altitude for the day and leave it fixed.

Such a user might take the instrument out a dozen times in the course of the day, quite possibly in different locations, and it is necessary only to follow the golden rule to tell the time. No ordinary user would think to keep a record of the solar azimuth at each observation but anyone who does would be in for a surprise. This is a consequence of an inherent asymmetry which can be a distraction when trying to understand the instrument's behaviour.

Fig. 5 shows one way of recording the observations. In each little diagram *south* is at the top, and the quadrant and the supporting disc are seen in plan (which makes the sloping edge of the quadrant appear foreshortened). Only four hour lines are shown, those for 0°, 30°, 60° and 90°. The position of the sun (strictly its azimuth) is also indicated.

Each row illustrates the orientations of the instrument at the even-numbered daylight unequal hours from hour 0 (sunrise) to hour 12 (sunset). At hours 0, 2, 4 and 6 the sun aligns with hour lines 0°, 30°, 60° and 90° respectively. At

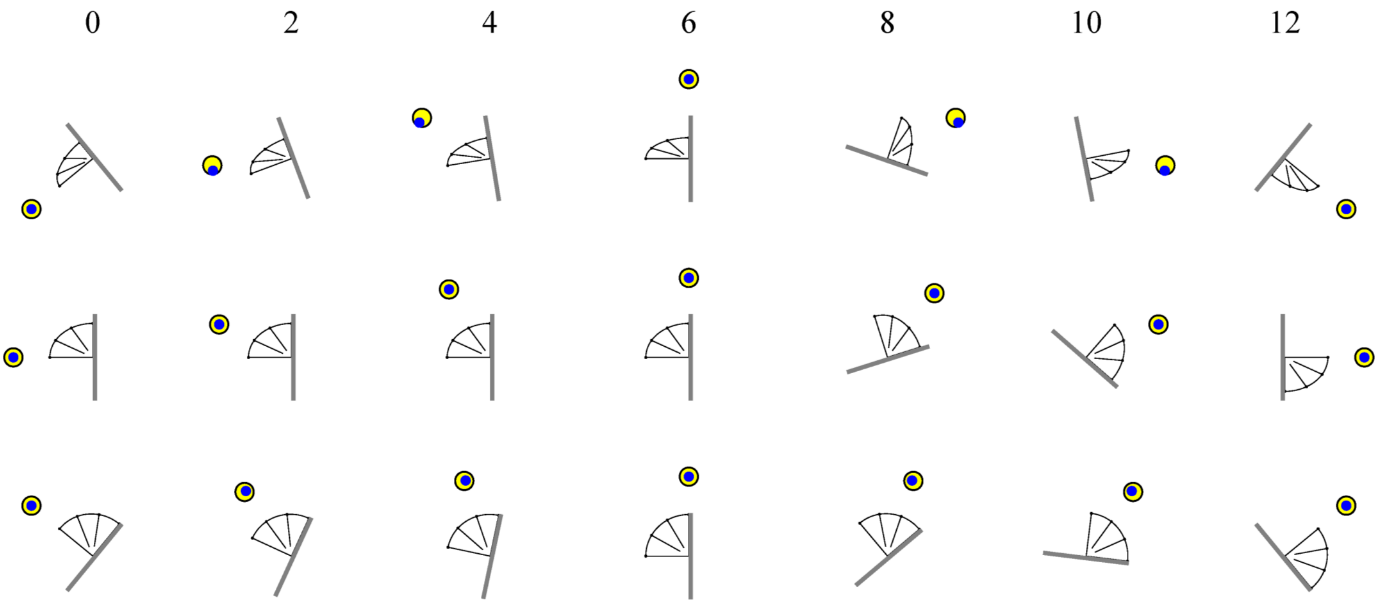

Fig. 5. Each of the three rows of figures shows plan views of the instrument (with south at the top) when making observations at the unequal hours 0, 2,…12. In each case, a schematic sun is shown which aligns with the relevant hour line; a blue dot indicates the actual azimuth of the sun at the unequal hour in question. In all cases the local latitude is 52° north. The solar declinations in the three rows are +23°, 0° and –23° so the noon altitudes (and quadrant slopes) are 61°, 38° and 15°.

hours 8, 10 and 12 the sun aligns with hour lines 60°, 30° and 0° respectively.

The three rows of figures depict the instrument at three different settings. In the top row, the slope is 61°, in the middle row it is 38°, and in the bottom row it is 15°.

Asymmetry is immediately apparent. In the middle row, the orientation of the instrument is fixed all morning but it is rotated 180° clockwise in the afternoon. In the top row the instrument is rotated in the morning but not by very much whereas in the afternoon it is rotated well over 180°. In the bottom row the instrument is rotated anti-clockwise in the morning but clockwise in the afternoon. Why?

The positions of the schematic suns give some clues. In the middle row the sun rises due east (on the left) and sets due west (on the right). This is the day of an equinox when the sun travels along the celestial equator. Given that the solar altitude at noon is 38°, we may infer that the local latitude is 52° north (90°–38°).

In the morning, the quadrant is fixed in the plane of the celestial equator but this cannot continue after noon. At hour 8, the instrument has to be rotated so that the 60° hour line aligns with the line to the sun. At hour 12 (sunset) the instrument has to be rotated so that the 0° hour line is due west. This accounts for the 180° afternoon rotation.

The same latitude, 52° north, also applies to the top and bottom rows; the noon solar altitudes are 61° and 15° so we may infer that the declinations are +23° and –23°, close to the summer and winter solstices. In the top row, the sun rises well to the north of due east and sets well to the north of due west. In the bottom row the sun rises well to the south of due east and sets well to the south of due west.

In each diagram, the blue spot indicates the actual solar azimuth at the unequal hour in question. At sunrise, noon and sunset, and also throughout the day of an equinox, the blue spot exactly aligns with the schematic sun. In most other cases the alignment is fairly close. The largest errors are at the summer solstice.

The instrument ranks as a universal sundial. It can certainly be used south of the equator. Fig. 6 shows three more rows of diagrams and they all apply to the southern hemisphere. South is still at the top but the sun is due *north* at noon.

In each case the noon solar altitude is 61°. The latitudes are 6° south in the top row, 29° south in the middle row and 52° south in the bottom row. The declinations this time are +23° (top row), 0° (middle row) and –23° (bottom row).

The middle row again depicts the day of an equinox with the sun rising due east and setting due west but notice that the instrument is rotated in the morning and held steady in the afternoon. The blue spot exactly aligns with the schematic sun throughout the day.

The top row relates to 6° south (in the tropics); the sun rises to the north of due east and sets to the north of due west. The instrument is rotated nearly 180° anti-clockwise in the morning; it is rotated clockwise in the afternoon but not by very much. The blue spot alignment is almost perfect.

The bottom row relates to a day close to the summer solstice at 52° south and should be compared with the top row in Fig. 5. The row of diagrams is identical to the top row in Fig. 5 turned upside down! The instrument is rotated well over 180° anti-clockwise in the morning and continues being rotated anti-clockwise in the afternoons but not by very much. The blue spot alignment is not very good.

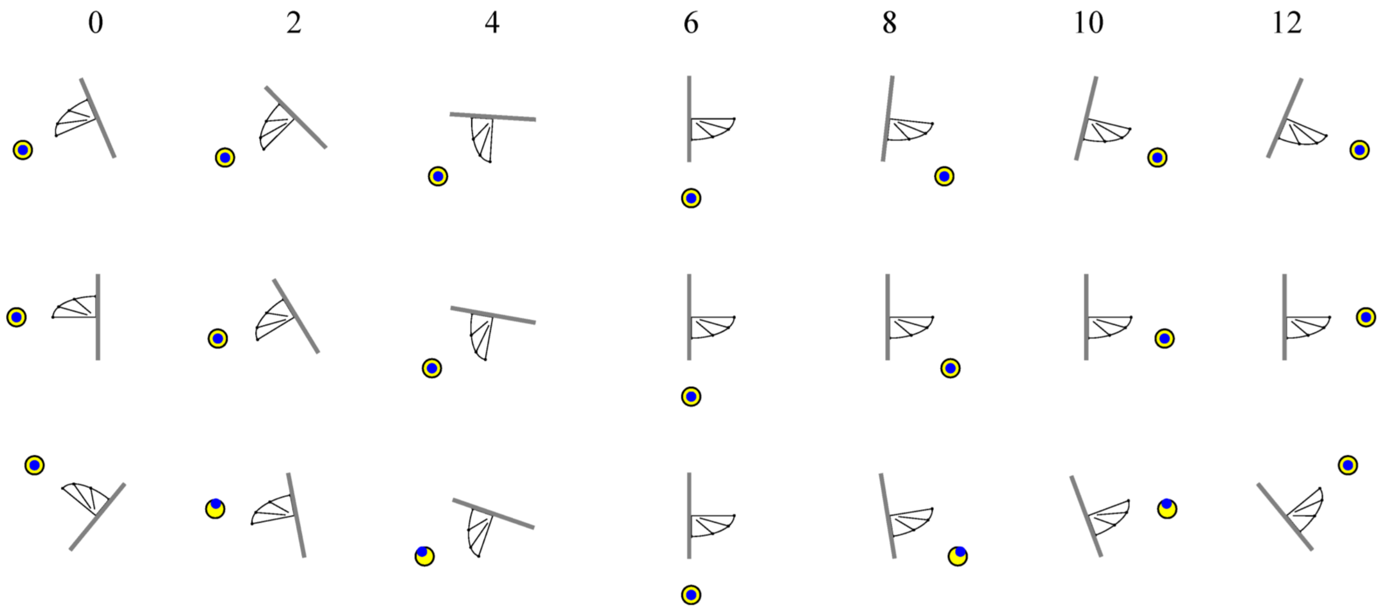

Fig. 6. Another set of plan views of the instrument (again with south at the top) when making observations at the unequal hours 0, 2,…12. The latitudes of the observer in the three rows are 6° south, 29° south and 52° south and the sun is due north at noon. The solar declinations are +23°, 0° and –23° respectively. Accordingly, in each case, the noon altitude (and quadrant slope) is 61°.

Determining the Altitude at Noon – Method I

The user of a Roman portable dial needed to know how to set the gnomon assembly to the correct slope. The supporting discs incorporate two scales (see Fig. 7) which together suggest that the designer took the solar altitude at noon a_N to be:

$$a_N = 90° - latitude + solar\ declination$$

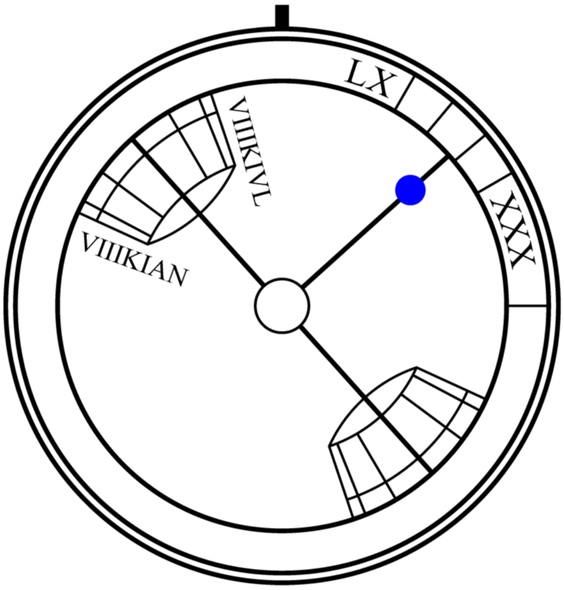

Fig. 7. The supporting discs with the top of the outer disc marked by the bracket for the suspension cord. The outer disc incorporates a latitude scale and the inner disc has a pair of declination scales.

The outer disc has four tick marks between the labels XXX and LX. The tick marks are at 30°, 40°, 50° and 60° anti-clockwise round from a horizontal reference tick mark and, together, they constitute a latitude scale.

The inner disc is rather richer in markings and in Fig. 7 these are dominated by a diameter and a radius which together form a T-shape. The inner disc can be rotated by means of the knob on the tail of the T. In Fig. 7 the tail has been set to a position between the 40° and 50° tick marks to indicate a latitude of 42°, the latitude of Rome.

Given that the cross-bar of the T is at right angles to the tail, the slope of the cross-bar is 90°–42° or 48°. The T-shape thus provides a simple mechanical way of determining the co-latitude, 90°–*latitude*, which, on the day of an equinox, is the solar altitude at noon.

At an equinox, the gnomon assembly should be set to align with the cross-bar of the T. At other times of year the gnomon assembly should be offset from the cross-bar by an angle which matches the solar declination.

Each end of the cross-bar is flanked on each side by three radial lines. One of the outer lines is labelled VIIIKIAN which rather cryptically refers to the winter solstice, the first point of Capricorn. At the other extreme, the summer solstice, the first point of Cancer, is labelled VIIIKIVL. The labels refer to eight days (VIII) before the Kalends (K) of, respectively, January (IAN) and July (IVL). The in-between lines refer to the first points of the other signs of the Zodiac, the lines indicating the solar declinations at these first points.

We have a simple circular slide rule. By setting the tail of the T to the local latitude, the slope of the cross-bar indicates the co-latitude and, by off-setting the gnomon assembly relative to the cross-bar by the declination, we complete the formula and set the slope of the gnomon assembly to the noon altitude of the sun.

The user was expected to know the local latitude and the position of the sun in the ecliptic. To help with the former, a list of 30 places is shown on the reverse side of the supporting disc together with the latitude of each; ROMA is shown as having latitude XLII. To attend to the latter, the user needed to know the dates of the first points.

If it happens to be the day of the summer solstice then the slope of the gnomon assembly should be set so that it aligns with the VIIIKIVL line. At the winter solstice, the slope of the gnomon assembly should be set so that it aligns with the VIIIKIAN line.

There is no compelling reason why the latitude range should be so limited. Rohr suggests that the range reflects the outermost limits of the Roman Empire. Certainly the latitudes of the 30 places listed on the reverse are all within this range.

It is also possible that the designer realised that the formula doesn't straightforwardly hold at low latitudes. For example, if the latitude is 20° and the declination is 23° the formula gives a noon altitude of 93° which is greater than the 90° maximum! This is not really a problem. If you set the slope of the gnomon assembly to 93° it will slope at 87° to the horizontal reference tick mark and this is the noon altitude. The assembly will now slope from top right to bottom left but that is of no consequence.

The designer might also have been wary of southern hemisphere latitudes. At latitude –50° at an equinox, the formula gives a noon altitude of 140° but the circular slide rule will again give a sensible result. The cross-bar will slope at 40° to the horizontal which is the noon altitude. The VIIIKIVL line will now be *below* the end of the cross bar so at the June solstice the slope of the gnomon assembly would be reduced. This is correct because at latitude –50° the June solstice is the *winter* solstice.

The latitude scale could go all the way from –90° to +90° and the two-step procedure for setting the slope of the gnomon assembly would always set it to the noon altitude of the sun.

The only intractable difficulty is that in very high latitudes (in the polar regions) there are times of year when the sun is below the horizon all day or above the horizon all day. With no sunrise or sunset, the whole concept of unequal hours fails and attempts to use the instrument will lead to nonsense results.

Determining the Altitude at Noon – Method II

A naive user who doesn't know the local latitude and doesn't know where the sun is in the ecliptic can nevertheless set the gnomon assembly to the correct slope!

The user should take the instrument out in the morning and align it so that the sun is in the plane of the supporting discs instead of in the plane of the gnomon assembly. Twist the suspension cord so that the sun shines equally on both the vertical faces. Then, keeping the support so aligned, adjust the gnomon assembly so that the sun is in its plane too.

This is most unlikely to give the correct slope and the procedure should be repeated at intervals. During the morning, the slope will gradually be increased. At noon it will be correct and it won't matter much if the exact moment of noon is missed. The altitude of the sun generally changes fairly slowly around noon. As soon as the slope seems to need decreasing, the user will know that the sun has passed its highest point and the gnomon assembly should be left alone. The instrument can then be used in the afternoon in the normal way and the slope will usually hold good for the morning of the following day too.

The user will need to make daily checks around noon. If the slope is too low, the sun will shine on the upper surface of the gnomon assembly even if the supporting disc is aligned with the sun. If the slope is too high, the indicated time will never reach hour 6.

This approach[5] works in both the northern and southern hemispheres. It makes no use of latitude or declination or of time of year, and dispenses with the need for any scales other than that on the gnomon assembly. It certainly dispenses with the need for an inner disc. A much cheaper model could therefore have been marketed which was easier to manufacture, very simple to set, and just as good at indicating unequal hours!

Underlying Theory – Preliminary Error Analysis

Fig. 8 shows a simplified version of Fig. 3. The quadrant has just three hour lines: the 0° line and the 90° line and one in between which has an angular offset of $15u°$ to the reference 0° line. For the moment u is assumed to be in the range 0 to 6, a morning unequal hour.

It is further assumed that the noon altitude is $a_N°$ and that this is the slope of the quadrant. The 90° line therefore makes an angle of $90–a_N°$ to the vertical, as shown. Finally assume that the quadrant has unit radius; this is why the $15u°$ line is shown with length 1.

Now suppose that a_u is the slope of the $15u°$ line relative to the horizontal X–Y plane; a_u is the altitude of the point in the sky seen by the bug when it sights along the $15u°$ line. From the instrument's point of view, at unequal hour u the solar altitude is a_u whatever the latitude and declination.

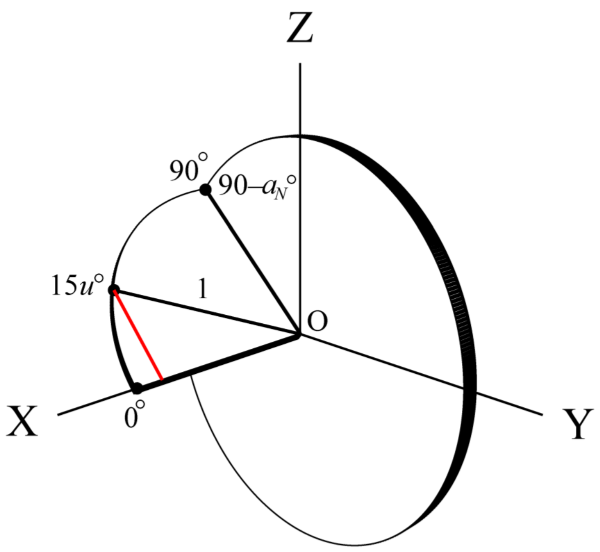

Fig. 8. In this variant of Fig. 3, the slope of the quadrant is $a_N°$, the solar altitude at noon. The hour line appropriate for unequal hour u is also drawn; it is at an angle 15u° to the reference 0° line.

The next step in the analysis is to determine the slope a_u in terms of u and a_N and then assess how close this value is to the actual altitude of the sun at unequal hour u.

Taking a_u as the slope of the $15u°$ line, and the length of the line as 1, the height of the outer end of the $15u°$ line above the X–Y plane is $\sin(a_u)$.

Now note the red line in Fig. 8. This is the perpendicular from the outer end of the $15u°$ line to the 0° line and its length is $\sin(15u)$. Moreover, the red line is parallel to the 90° line so its slope (relative to the X–Y plane) is a_N. Accordingly, the height of the outer end of the $15u°$ line above the X–Y plane is $\sin(15u).\sin(a_N)$.

These two values for the height can be equated:

$$\sin(a_u) = \sin(15u).\sin(a_N)$$

This relationship is well known. For example, exactly the same relationship underlies the design of the horary quadrant.[6]

Given the noon altitude a_N the relationship enables us to determine a solar altitude a_u at any unequal hour u. The sine function takes care of the afternoon hours so there is no need to subtract u from 12. The result may not be quite the correct altitude but the error is normally sufficiently small that it can be ignored. Let's investigate the errors…

Clearly if $u=0$ or $u=12$ then $a_u=0$ and if $u=6$ then $a_u=a_N$ so the error is always zero at sunrise, noon and sunset. It can also be shown that the error is always zero at the equator (latitude 0°) whatever the time of year and is always zero at an equinox (declination 0°) whatever the latitude. In all other circumstances the relationship will give a solar altitude that is not quite correct.

For example, Fig. 2 illustrated a special case in which $a_N=90°$, so the relationship simplifies to $a_u=15u$ implying that when $u=2$ the altitude is 30°. This is true on the equator at an equinox but the noon altitude can be 90° anywhere in the tropics, for example at latitude 23° on a day when the declination is 23°. There, at unequal hour 2, the altitude of the sun is actually just over 29° and not 30°.

Errors – More General Considerations

In general terms, the errors are always zero at the equator and become greater as the latitude increases.

The error can be measured in three ways. An error in altitude means that there will be an error in azimuth (as illustrated in Figs 5 and 6) and an error in the indicated time. Since the error in altitude is the source of the other two errors, it merits particular attention.

Consider the range of latitudes where the noon solar altitude can be 46°. The range extends from latitude 21° (when the declination is −23°) via latitude 44° (at equinoxes, when the declination is 0°) to latitude 67° (when the declination is +23°).

In all cases the gnomon assembly has to be set so that it slopes at 46° to the horizontal and the expression for $\sin(a_u)$ gives rise to the red curve in Fig. 9. This is exactly right for the mid-range latitude of 44° where 46° is the noon altitude at an equinox.

At the tropical end of the range, latitude 21°, the correct *altitude-versus-u* relationship is shown by the top black curve in Fig. 9. This almost coincides with the red curve.

The second black curve, immediately below the red curve, applies to a British latitude 52° and again there is minimal error. The third black curve down applies to latitude 62° and the errors are starting to be significant.

The bottom black curve applies to latitude 67°, just inside the Arctic circle. The errors are large and, informally, it is easy to see why. The red curve assumed by the instrument shows the altitude climbing steadily at roughly 10° per (unequal) hour for four hours and then levelling off. There is a flat peak around noon. There is also a flat bottom around midnight but, since the sun is usually out of sight then, this is not generally noticed. In the circumstances of the bottom curve, there are (just) 24 hours of daylight. The curve is noticeably more S-shaped and hence significantly diverges from the red curve.

There is nothing special about choosing 46° for the noon altitude. Other values would give similar curves in Fig. 9.

At a given latitude (north or south), the general rule is that there are no errors at an equinox and the errors increase with increasing daylight (as we approach the flat bottom). There is also an increase with *de*creasing daylight but very much less so because there is no tendency to an S-shape.

Errors – The Polar Limit Theorem

As noted, an altitude error translates into an azimuth error and an error in indicated time. In exploring a computer model of the Roman portable dial it was noted that at $u = 2$ the error in the bottom curve in Fig. 9 translated into exactly two (common) hours. Moreover this is true whenever *latitude* + *declination* = ±90°. Although this observation would have been of no interest to those who once used the instrument, the explanation is entertaining and worth exploring…

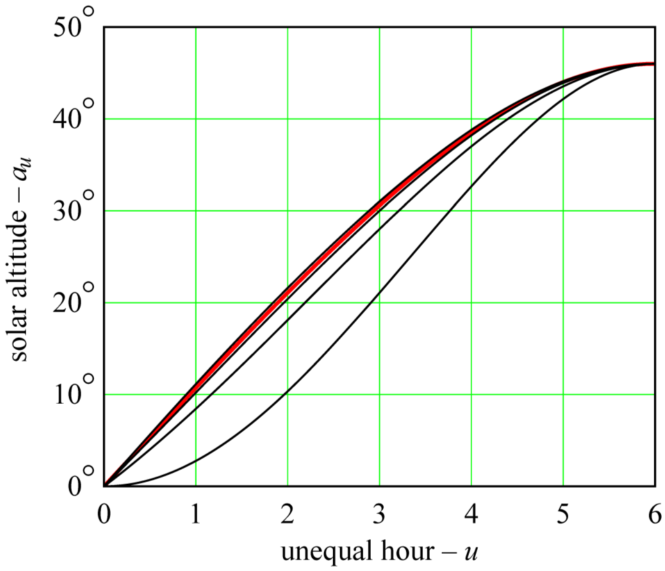

Fig. 9. Five examples where the noon altitude is 46°. The red curve shows the quadrant's understanding of the solar altitude for unequal hours from 0 to 6. At latitude 44°, on the day of an equinox, this exactly matches the actual solar altitude over this period. The lowest curve shows the solar altitude at latitude 67° on a day when the solar declination is 23°. The sun is on the horizon at midnight but otherwise above the horizon for all 24 hours of the day. The errors are large. The other curves are for latitudes 21°, 52° and 62°. The errors are small.

Take the latitude as ϕ and the declination as δ. At the threshold of 24-hour daylight $\phi+\delta = 90$. Now, noting that $a_N = 90-\phi+\delta$, we have two simultaneous equations which give $\phi = 90-a_N/2$ and $\delta = a_N/2$

Using the normal formula for solar altitude we can write:

$$\sin(a_u) = \sin(\phi).\sin(\delta) - \cos(\phi).\cos(\delta).\cos(H_{au})$$

Here H_{au} is the hour angle (since midnight) at which the sun has altitude a_u. Equating this to $\sin(15u).\sin(a_N)$ and substituting the expressions for ϕ and δ above leads to:

$$2\sin(15u).\sin(a_N) = \sin(a_N) - \sin(a_N).\cos(H_{au})$$

The three instances of $\sin(a_N)$ cancel and we are left with:

$$\cos(H_{au}) = 1-2\sin(15u)$$

This is a remarkable result. If the sum of the latitude and declination is ±90°, the hour angle when the instrument indicates unequal hour u is independent of latitude, declination and noon solar altitude. The hour angle at unequal hour u is actually $30u$ (24-hour daylight means 30° per unequal hour). The result applies only in the polar regions but theoretically it holds more generally, such as at latitude 52° when the solar declination is (impossibly) 38°.

I call this the Polar Limit Theorem since it applies in the polar regions when the declination is on the threshold that gives 24-hour daylight. If $u = 2$ then $\cos(H_{au}) = 0$ and $H_{au} = 90°$. But with 24-hour daylight the actual hour angle at hour 2 is 60°. The error equates to a difference in hour angle of 30° which is two common hours or one unequal hour. We may guess that the designer did not know this!

Conclusions

The foregoing is presented primarily as a guide to how the instrument is used and how it works both in the mornings and in the afternoons. For a historical context see Rohr[1] and Wright[2]. Neither says who might have owned and used such instruments but both note that a few similar dials have been found and that the design was used by Greeks and Romans. Neither suggests that the design has predecessors or successors and here is some speculation…

A large-scale version of the gnomon assembly could be used as a hand-held instrument. Without the supporting disc, it would be difficult to hold the gnomon assembly at the correct angle but, in principle, it would be possible. As a stand-alone instrument, the gnomon assembly is a little like an ancient L-shaped stick dial but with the longer leg curved instead of straight. Could such sticks have been the inspiration that led to the design of the gnomon assembly?

Noting that the horary quadrant shares the same underlying mathematics, could there be some conceptual link there?

The horary quadrant was simpler to make, was as easy to use, did not have a gnomon assembly to break off or lose, and did not swing around in the breeze at the end of a length of cord. On the other hand, the hour lines on an horary quadrant bunch together when the noon altitude is low and the instrument becomes harder to read.

APPENDIX
Mathematical Relationships

Readers who wish to set up and explore a computer model can easily achieve this with a simple spreadsheet. Just five relationships are needed to determine the error in indicated time:

$\cos(H_{sr}) = \tan(\phi).\tan(\delta)$: H_{sr} is the hour angle of sunrise (in degrees since midnight) given latitude ϕ and solar declination δ.

$H_u = H_{sr}+u(180-H_{sr})/6$: H_u is the hour angle of the sun at unequal hour u.

$\sin(a_u) = \sin(15u).\sin(a_N)$: a_u is the solar altitude when the instrument *indicates* unequal hour u; the instrument is set to indicate a solar altitude of a_N at noon.

$\cos(H_{au}) = (\sin(\phi).\sin(\delta)-\sin(a_u))/(\cos(\phi).\cos(\delta))$: H_{au} is the hour angle of the sun when the solar altitude is a_u. If u>6 then subtract the value of H_{au} from 360.

$E_{HA} = H_u-H_{au}$: E_{HA} is the difference between the hour angle at u and the hour angle when the instrument indicates u; $4E_{HA}$ is the error in (common) minutes.[7]

To determine the errors in solar altitude and solar azimuth further relationships are required:

$\sin(a_{Hu}) = \sin(\phi).\sin(\delta)-\cos(\phi).\cos(\delta).\cos(H_u)$: a_{Hu} is the altitude when the hour angle is H_u. The altitude when the instrument indicates hour u is a_u.

$E_{alt} = a_{Hu}-a_u$: E_{alt} is the difference between the altitude at u and the altitude when the instrument indicates u.

$\tan(A_u) = (\cos(\delta).\sin(H_u))/(\cos(\phi).\sin(\delta)+\sin(\phi).\cos(\delta).\cos(H_u))$: A_u is the azimuth (measured clockwise round from due north) when the hour angle is H_u. If u>6 then add 360 to the value of A_u.

A_{Hau}, the azimuth when the hour angle is H_{au}, is determined likewise but use H_{au} instead of H_u. If u>6 then add 360 to the value of A_{Hau}.

$E_{az} = A_u-A_{Hau}$: E_{az} is the difference between the azimuth at u and the azimuth when the instrument indicates u.

When the instrument indicates hour u it will, in general, be rotated from its rest position (the disc facing due east) and two final relationships are required:

$\tan(O_u) = \sin(15u).\cos(a_N)/\cos(15u)$: O_u is the offset, *in plan*, of hour line u from the reference 0° hour line. If u>6 then u must be replaced by $12-u$ in the expression.

$R_u = A_{Hau}-O_u-90°$: R_u is the angle by which the instrument is rotated from its rest position (the discs facing due east) when the instrument indicates hour u.

REFERENCES and NOTES

1. The instrument is described at: *www.mhs.ox.ac.uk/collections/imu-search-page/record-details/?thumbnails=on&irn=3237&TitInventoryNo=51358*
2. R.R-J. Rohr: 'Portable sundials in ancient Rome', *BSS Bulletin,* 94.2, 42–45 (June 1994).
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7. Wright (Ref. 3, page 183) gives a graphical representation of the errors but expresses these in unequal minutes (1/60 of an unequal hour). To reproduce his results scaling is required.

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